



Distance Measurement using an Infrared Distance Sensor Based on Lagrange and Newton Interpolating Polynomials

Rawaz H. Abdullah

Dept. of Communication Engineering, Technical College of Engineering, Sulaimani Polytechnic University, Kurdistan Region, Sulaimani, Iraq.

E-mail: rawaz.abdullah@spu.edu.iq

Article info

Original: 20 Feb. 2015
Revised: 8 Apr. 2015
Accepted: 19 Apr. 2015
Published online:
20 Sep. 2015

Key Words:

Infrared (IR) distance measuring sensor, distance measurement, optical triangulation method, Newton and Lagrange interpolating polynomials, error estimation.

Abstract

This paper presents two models for distance measurement to a very popular infrared (IR) distance measuring sensor from SHARP, which has applications in projects that require cheap and accurate distance measurements. The sensor outputs a voltage at any distance (with a range from 10 cm to 80 cm) from an object based on the optical triangulation method. The variety of the reflectivity of the object, the environmental temperature and the operating duration are not influenced easily to the distance detection. The presented models in this paper are based on Newton and Lagrange interpolating polynomials, which require a few data points to compute the polynomial equations and thereby the distance from any object could be computed easily. The expected errors in distance estimation for the two proposed models are analyzed. As it is evident from the results, the two models could be used easily with insignificant amount of error.

Introduction

In general, infrared (IR) sensors are broadly used as proximity sensors, basically for collision and obstacle avoidance in robotics. Compared to ultrasonic sensors, IR sensors are cheaper and have faster time response. However, they have non-linear characteristics and they depend on the reflectance properties of the object surfaces, which limits their applications for ranging purposes [1][2][3].

Some previous related works that used IR sensors for distance measurement were based on Phong illumination model. The model is able to approximate the reflectance properties of any surface illuminated by a point light source. In [4] the Phong model is used for distance measurement in a range of 5 cm to 23 cm. The model is used to determine two coefficients. To utilize the model in unknown environments, the distance

to obstacles must come from other sensors such as sonar or ultrasonic sensors. Another technique based on the Phong model is described in [2]; the technique uses a single infrared sensor to measure distance from 10 cm to 24 cm. The sensor is mounted at the front of a robot which rotates to perform two angular scans to obtain the model coefficients. The phong model is also used in [3] to measure distance in a range between 2 cm and 14 cm. In [1] a new IR sensor is described based on light intensity back-scattered from objects to measure distances up to 1 m. The sensor output follows the photometry inverse square law. Although the model needs only one coefficient, the coefficient to be estimated requires the use of another sensor.

The main drawback of the previous works is the high dependence of the reflective properties of the object surface. A more efficient technique based on the optical triangulation method has been used for distance measurement. This method is highly independent from optical surface properties and allows significant and more accurate range measurements. Triangulation methods are based on the projection of a light pattern onto a surface and imaging it by a photo detector or camera, as shown in Fig. 1. A beam from a light emitting diode (LED) or a laser is directed through optical lenses toward the object. The bundling of the reflected light is done by another lens and passed to a photo detector element. The photo detector can be either a position sensitive detector (PSD) or a charge coupled device (CCD). PSD is a special type of photo detector used to measure the position of a light spot. Depending on the location of the object, the reflected light produces differences in the electric current on the light sensitive area of the PSD. On the basis of this difference, the position or distance of the object can be measured [5][6]. In [7] and [8] the triangulation method is used for height measurements on water surfaces. More recently, in [6] an IR sensor based on the triangulation method is used to measure distances up to 35 cm. The effects of different color objects on the sensor output voltage is studied.

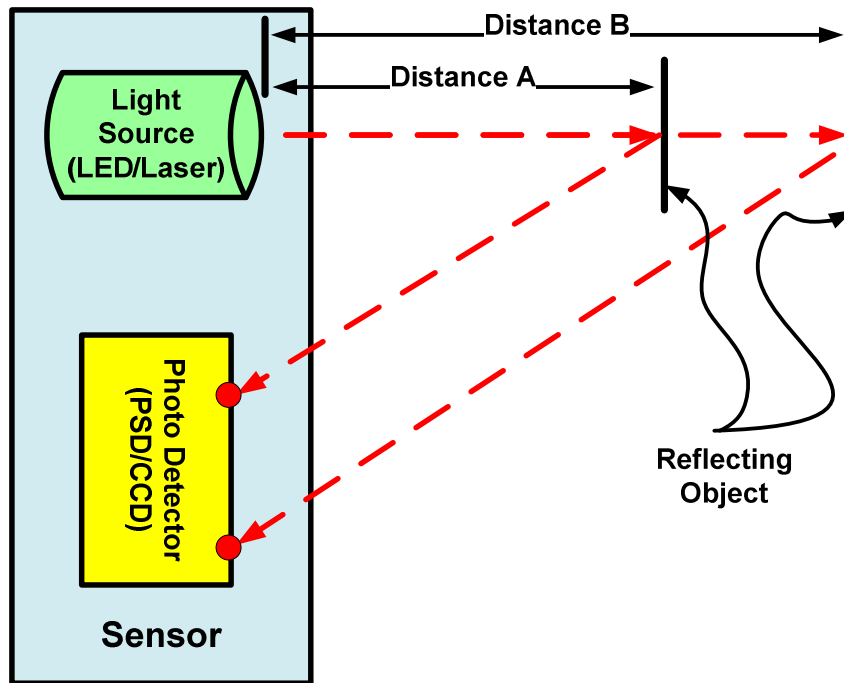


Fig.1 Principle of optical triangulation method

The IR distance measuring sensor used in this work is GP2Y0A21YK0F from SHARP Corporation [9], which is more economical than sonar or ultrasonic rangefinders, yet it provides much better performance than other IR alternatives [10]. The sensor measures distances based on the triangulation principle. Therefore, the variety of the reflectivity of the object, the environmental temperature and the operating duration are not influenced easily to the distance detection because of adopting the triangulation method [9]. For distance estimation, stochastic modeling of the sensor is presented in [10], but in this paper Lagrange and Newton

interpolation polynomials are used to model the sensor which require a few data points to compute the polynomial equations and thereby the distance from any object could be computed easily.

Sharp GP2Y0A21YK0F Infrared Sensor

The IR sensor has three terminals, as is shown in Fig.2. The terminals from left to right are output voltage (V_o), ground (GND), and supply voltage (V_{cc}). The device outputs voltage corresponding to the detection distance. The sensor uses a 3-pin Japan solder-less terminal (JST) connector [9].



Fig.2 Sharp infrared distance sensor

As shown in Fig.3, the sensor unit (GP2Y0A21YK0F) is composed of an integrated combination of three main components: position sensitive detector (PSD), infrared light emitting diode (LED) with its driving circuit, and signal processing circuit [9].

Fig.3 Internal block diagram of the sharp infrared sensor

For distance measurement, after connecting the supply voltage, pulses of infrared light (with a wavelength of $870\text{nm} \pm 70\text{nm}$) are emitted by the LED and then reflected back by the object to the PSD. PSDs are commonly used in non-contact distance measurements using the triangulation principle. The detected light comes back at an angle that is dependent on the distance of the reflecting object. The

signal processing circuit uses the triangulation method to detect the reflected beam angle. By knowing the angle, the output circuit generates a unique voltage at any specific distance from the object [9].

Experimental Procedure and Practical Results

In order to model the distance sensor it is necessary to collect a considerable amount of data. Distance measuring range of the tested GP2Y0A21YK0F sensor, as given by the manufacturer, is about 10 cm to 80 cm [9][10].

The output terminal voltage of the sensor can be connected to an analog to digital converter (ADC) port of a microcontroller for taking distance measurements. In this work, the IR sensor output terminal is interfaced to one of the ADC ports of Arduino UNO microcontroller board with 10-bit resolution to obtain the output voltage of the sensor. As recommend by the device manufacture, a by-pass capacitor of $10\mu\text{F}$ is inserted between the supply voltage and ground terminals of the sensor to stabilize power supply line. The schematic diagram of the interface is shown in Fig. 4.

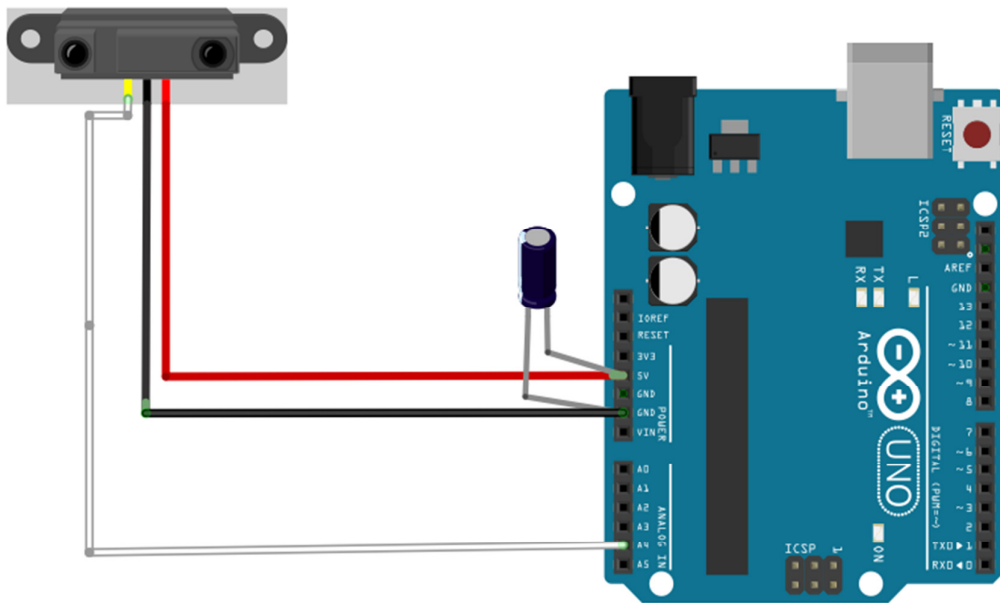


Fig.4 The schematic diagram of the IR sensor interfaced to an Arduino UNO board

The Arduino board is connected to a computer by a universal serial bus (USB). The board is programmed using a processing-based integrated development environment (IDE), which is similar to C++ with some slight simplifications and modifications. A tape-measure is used to measure the actual value of distance. At any specific distance from the sensor, the generated voltage at the output terminal of the sensor (V_o) is continuously reported by the Arduino board and sent to the computer. The experimental setup is displayed in Fig. 5. As it can be seen, the sensor is mounted on a flat surface to obtain accurate measurement during the data collection procedure. The sensor was moved in a right angle towards the target from a distance of 10 cm to 80 cm, with intervals of 1 cm.

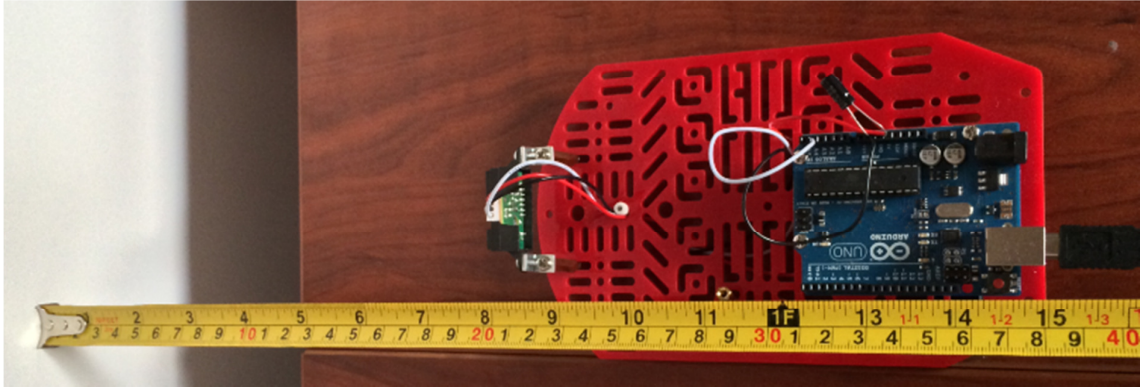


Fig.5 Experimental setup

To get better results, during each reading from the sensor, ten samples were read and then the average of these sample values was taken. The resulting recorded data (the average generated voltage by the sensor as a function of distance) is plotted using MATLAB, as shown in Fig. 6. It can be noticed that the measured data are quite similar to the data given by SHARP datasheet [4]. As the sensor moves farther from the object, the IR output voltage decreases non-linearly. This means that as the distance increases linearly, the analog output voltage of the sensor decreases non-linearly. To compute distance from the sensor output voltage, it is necessary to create a representative mathematical equation (model) that relates the output voltage with distance, as discussed latter.

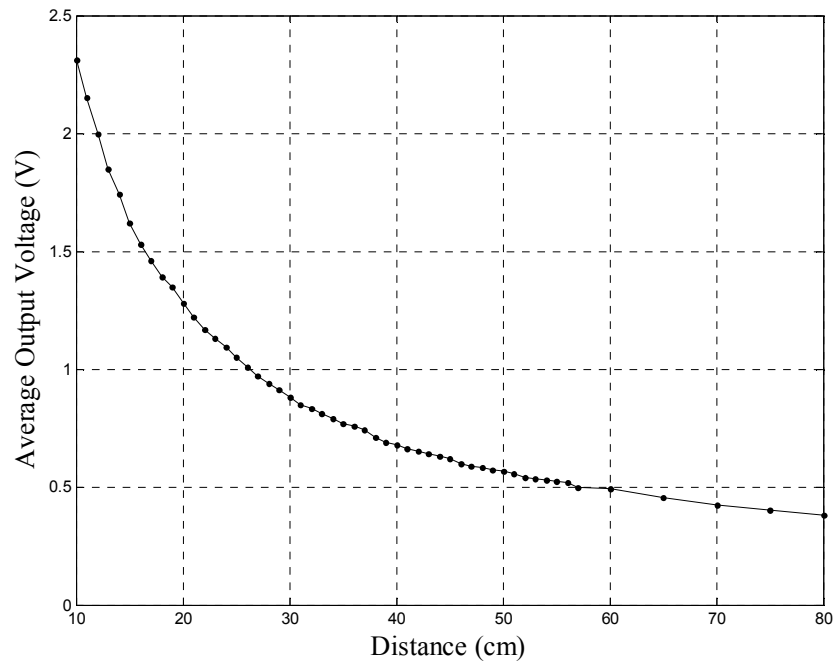


Fig. 6 Plot of the output voltage obtained from the IR sensor as a function of distance

Modeling the IR Distance Sensor

To model the sensor, i.e. to find a relationship between the generated voltage and the distance, Lagrange and Newton interpolating polynomials are used in this paper. As given in Table-I, six data points¹ that obtained from the practical measurements (Fig.6) are used to implement the polynomial equations.

Table-I: Data Points

Average Output Voltage (V)	Distance (cm)
2.31	10
2	12
1.46	17
1.095	24
0.71	38
0.38	80

A. Using Lagrange Interpolating Polynomial

The general n-th order Lagrange interpolation polynomial based on (n + 1) data points is given by [11][12]:

$$f_{Lagrange}(x) = \sum_{i=1}^{n+1} f_{Lagrange}(x_i) L_i(x) \quad (1)$$

Where,

$$L_i(x) = \prod_{\substack{j=1 \\ j \neq i}}^{n+1} \left(\frac{x - x_j}{x_i - x_j} \right) \quad (2)$$

To evaluate or estimate the distance based on the generated voltage of the IR sensor, one needs to represent the distance to be computed as a function of the output voltage of the IR sensor based on the 6 data points of Table-I. Substituting the data points into (1) and (2), we can obtain a fifth-order Lagrange polynomial as:

$$f_{Lagrange}(x) = \sum_{i=1}^6 f_{Lagrange}(x_i) L_i(x) \quad (3)$$

$$= 10L_1(x) + 12L_2(x) + 17L_3(x) + 24L_4(x) + 38L_5(x) + 80L_6(x)$$

Where,

$$L_1(x) = \frac{(x - 2)(x - 1.46)(x - 1.095)(x - 0.71)(x - 0.38)}{0.98863092} \quad (4)$$

$$L_2(x) = \frac{(x - 2.31)(x - 1.46)(x - 1.095)(x - 0.71)(x - 0.38)}{-0.3165984306} \quad (5)$$

$$L_3(x) = \frac{(x - 2.31)(x - 2)(x - 1.095)(x - 0.71)(x - 0.38)}{0.13570335} \quad (6)$$

$$L_4(x) = \frac{(x - 2.31)(x - 2)(x - 1.46)(x - 0.71)(x - 0.38)}{-0.110480210465625} \quad (7)$$

¹ The choice and the number of these non-equally spaced data points is tested to obtain the best curve fit. Reducing the number of data points will affect the accuracy of the distance to be computed compared to the true or test distance.

$$L_5 = \frac{(x - 2.31)(x - 2)(x - 1.46)(x - 1.095)(x - 0.38)}{0.1966734} \quad (8)$$

$$L_6 = \frac{(x - 2.31)(x - 2)(x - 1.46)(x - 1.095)(x - 0.71)}{-0.7967389716} \quad (9)$$

The value of x represents the output voltage of the IR sensor, and $f_{Lagrange}(x)$ is the distance to be estimated.

B. Using Newton Interpolating Polynomial

Newton's interpolating polynomial is among the most popular and useful forms of interpolating polynomials. The general equation n-th order Newton interpolation polynomial, which requires (n+1) data points, is given by [11][12]:

$$f_{Newton}(x) = b_1 + b_2(x - x_1) + \dots + b_{n+1}(x - x_1)(x - x_2) \dots (x - x_n) \quad (10)$$

Where b_1, b_2, \dots, b_{n+1} are the polynomial coefficients, and can be calculated from the (n+1) data points as [11][12]:

$$b_1 = f[x_1] \quad (11)$$

$$b_2 = f[x_2, x_1] \quad (12)$$

$$b_3 = f[x_3, x_2, x_1] \quad (13)$$

⋮

$$b_n = f[x_n, x_{n-1}, \dots, x_2, x_1] \quad (14)$$

The bracketed functions are divided differences. They can be represented generally as [11][12]:

$$f[x_i] = f_{Newton}(x_i) \quad (15)$$

$$f[x_i, x_j] = \frac{f_{Newton}(x_i) - f_{Newton}(x_j)}{x_i - x_j} \quad (16)$$

$$f[x_i, x_j, x_k] = \frac{f[x_i, x_j] - f[x_j, x_k]}{x_i - x_k} \quad (17)$$

$$f[x_n, x_{n-1}, \dots, x_2, x_1] = \frac{f[x_n, x_{n-1}, \dots, x_2] - f[x_{n-1}, x_{n-2}, \dots, x_1]}{x_n - x_1} \quad (18)$$

The data points ($n + 1 = 6$ or $n = 5$) given in Table-I can be used to compute the divided differences, by substituting their valued into equations (15), (16), (17), and (18):

$$f[x_i] = 10 \quad (19)$$

$$f[x_2, x_1] = -6.451612903226 \quad (20)$$

$$f[x_3, x_2, x_1] = 3.30311336004 \quad (21)$$

$$f[x_4, x_3, x_2, x_1] = -6.30198516853 \quad (22)$$

$$f[x_5, x_4, x_3, x_2, x_1] = 1.852948654038 \quad (23)$$

$$f[x_6, x_5, x_4, x_3, x_2, x_1] = -26.94370148872 \quad (24)$$

The results of equations (19) through (24) represent the coefficients $b_1, b_2, b_3, b_4, b_5,$ and $b_6,$ respectively. Substituting the data points of Table-I and the computed coefficients into (10), form a fifth-order Newton interpolating polynomial:

$$\begin{aligned}
 f_{Newton}(x) = & 10 - 6.451612903226(x - 2.31) \\
 & + 3.30311336004 (x - 2.31)(x - 2) \\
 & - 6.30198516853 (x - 2.31)(x - 2)(x - 1.46) \\
 & + 1.852948654038 (x - 2.31)(x - 2)(x - 1.46)(x - 1.095) \\
 & - 26.94370148872 (x - 2.31)(x - 2)(x - 1.46)(-1.095)(x - 0.71)
 \end{aligned} \tag{25}$$

In this case the value of x represents the output voltage of the IR sensor, and $f_{Newton}(x)$ is the distance to be estimated.

C. Distance Estimation

Either equation (3) or (25) can be used to obtain an estimate of the distance from the output voltage of the IR sensor. Any uncertainty or inaccurate output voltage of the sensor, due to noise or other sources of error will produce uncertainty in the distance to be estimated, therefore at each reading, the average of ten sample voltages are used in the process of distance estimation.

The output program for the distance measurement is shown in Fig.7. In this case the true distance is 30 cm. As it can be seen, the estimate distance based on the Lagrange polynomial is about 29.78cm, the estimate distance based on the Newton polynomial is also 29.78 cm. The absolute error in both cases is about 0.22 cm. Following the same procedure, the sensor is moved in direction of the object and the two equations are used to compute distance. Fig.8 shows the distance measurement based on the polynomial equations with comparisons to the true distances.

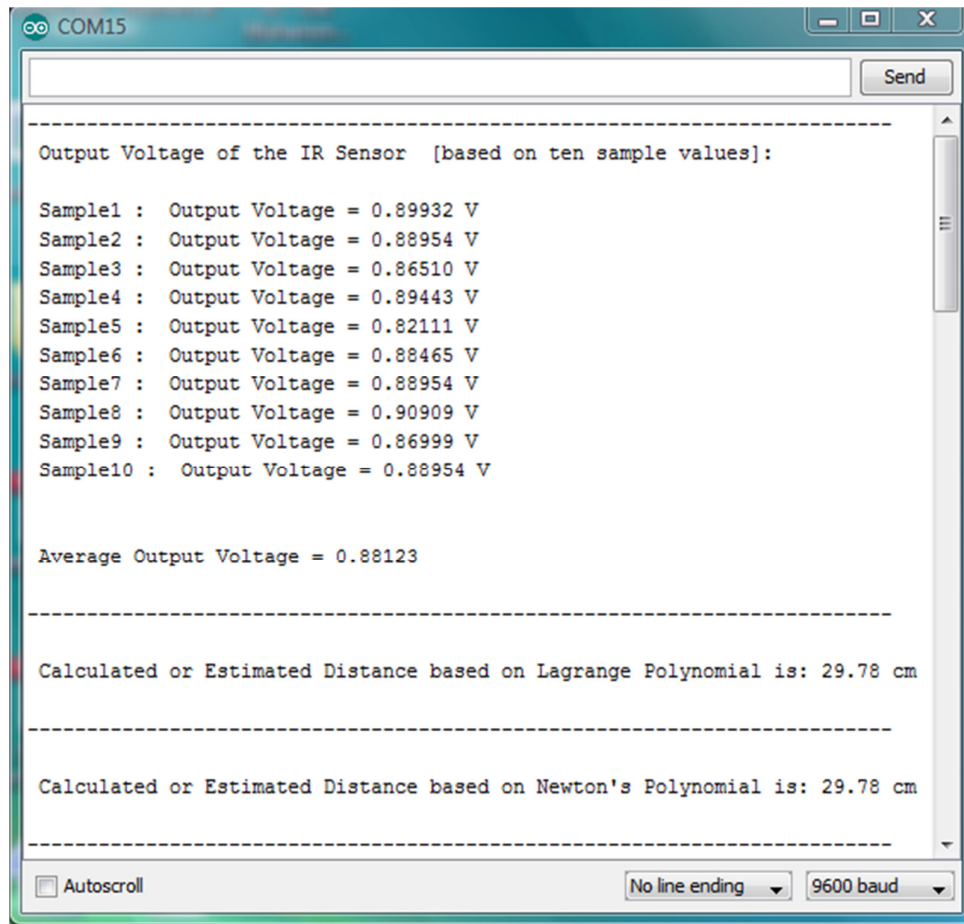


Fig.7 Output program for the distance measurement

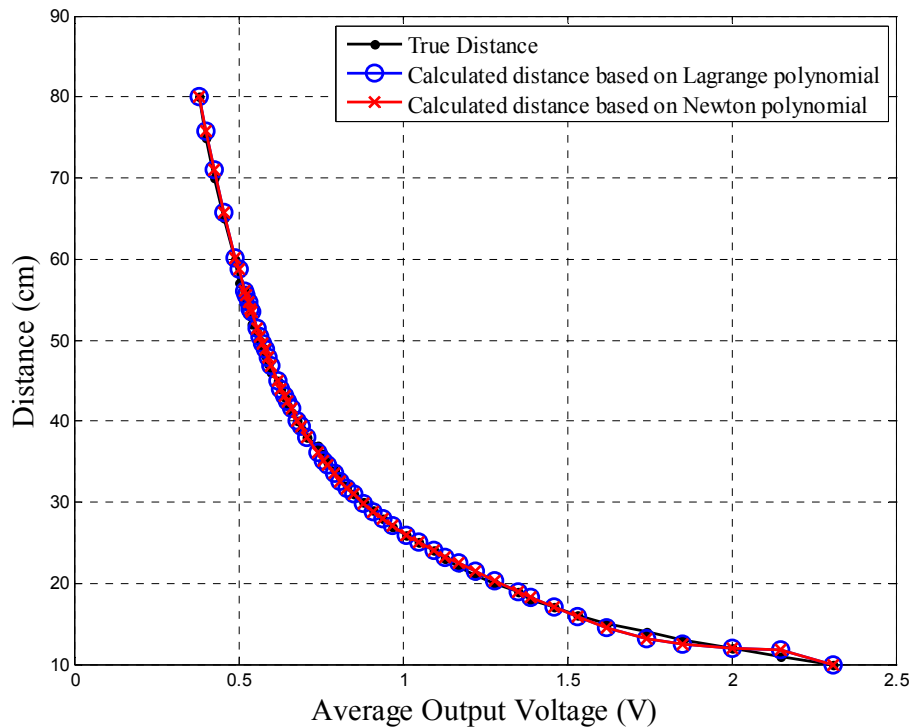


Fig. 8 Comparison of calculated and true distances

The absolute errors of the calculated distances compared to the true distances are shown in Fig.9. As it can be seen, the calculated distances using the Lagrange and Newton polynomials are quite the same and there is no significant difference or improvement in accuracy compared to the true distance, while in terms of computational complexity, the distance computation based on the Newton's polynomial is simpler and more preferred.

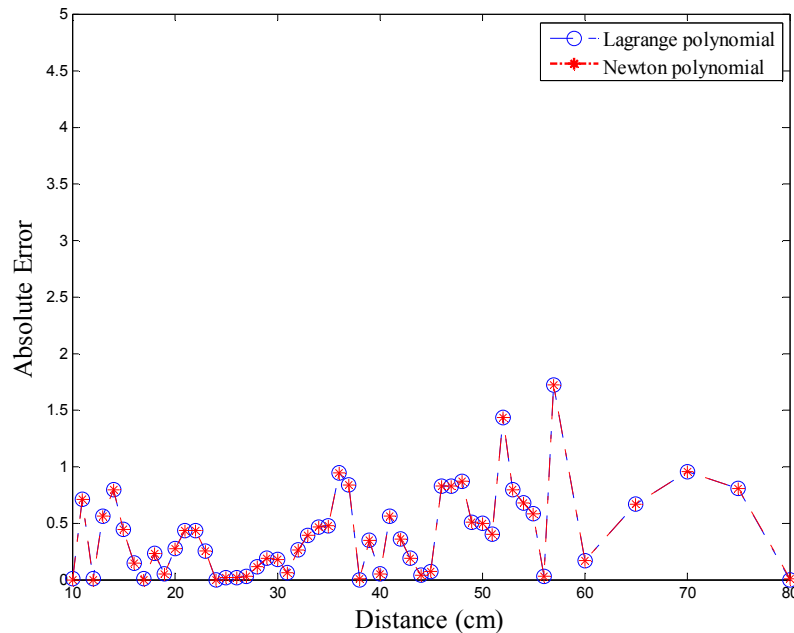


Fig. 9 Absolute errors of the calculated distances

Conclusion

In this work an infrared sensor from SHARP Corporation is used to compute distance between the sensor and other objects with a range from 10 cm to 80 cm (the sensor could be mounted to ensure the target is at least 10 cm away from the sensor). The sensor outputs voltage at any specific distance based on the triangulation method, which makes the sensor very popular and therefore the variety of the reflectivity of the object does not affect the accuracy of the sensor. For distance measurement, two models are proposed based on fifth-order Lagrange and Newton interpolating polynomials, which are given by (3) and (25), respectively. The two models use the output voltage of the sensor to compute distance; therefore to get better results and to reduce any uncertainty and noise in the output voltage, the average of ten sample voltages are used as a reference voltage. The models are compared and it can be concluded from the results, the accuracy of the two models are extremely similar. In general both of the models could be used but in terms of complexity the Newton polynomial is simpler and more desirable.

The two models used in this work are simple and based on only six data points, to obtain more accurate distance measurements, more data points could be used in future to implement higher order polynomials with best curve fit and minimum error. Other more complex interpolation methods such as cubic spline interpolation and least-square based regressions can be used in future to minimize error.

References

- [1] G. Benet, F. Blanes, J.E. Simó, P. Pérez, " Using infrared sensors for distance measurement in mobile robots " , Journall of Robotics and Autonomous Systems, Vol. 40, pp. (2555-266), 2002.
- [2] Miguel Angel Garcia and Agusti Solanas, " Automatic distance measurement and material characterization with infrared sensors " , Springer-Verlag Berlin Heidelberg, RoboCup 2004, Vol. 3276, pp. (451-458), 2005.

- [3] Tarek Mohammad., " Using ultrasonic and infrared sensors for distance measurement ", World Academy of Science, Engineering and Technology, Vol. 51, pp.(293-298), 2009.
- [4] P. M. Novotny and N. J. Ferrier, " Using infrared sensors and the Phong illumination model to measure distances ", IEEE International Conference on Robotics and Automation, pp.(1644-1649), 1999.
- [5] Henrik Andersson, "Position Sensitive Detectors - Device Technology and Applications in Spectroscopy", Doctoral Thesis, Department of Information Technology and Media, Mid Sweden University, 2008.
- [6] M.R. Yaacob, N.S.N. Anwar, and A.M. Kassim, " Effect of Glittering and Reflective Objects of Different Colors to the Output Voltage-Distance Characteristics of Sharp GP2D120 IR ", ACEEE Int. J. on Electrical and Power Engineering, Vol. 03, No. 02, 2012.
- [7] C. Mulsow, M. Schulze, P. Westfel, " An optical triangulation method for height measurements on instationary water surfaces ", IAPRS, Vol. XXXVI, Part 5, 2006.
- [8] Christian Mulsow, Hans-Gerd Maas, Patrick Westfeld, and Matthias Schulze, " Triangulation methods for height profile measurements on instationary water surfaces ", Journal of Applied Geodesy, Vol.2, pp.(21-29), 2008.
- [9] Data sheet of (GP2Y0A21YK0F) Sharp distance measuring sensor, Sheet No. E4-A00201 EN, ©SHARP Corporation, 2006. (http://www.sharpsma.com/webfm_send/1489)
- [10] Paulo Malheiros, José Gonçalves and Paulo Costa, " Towards a more accurate infrared distance sensor model ", International Symposium on Computational Intelligence for Engineering Systems, ISEP-Porto Portugal, 2009.
- [11] Steven C. Chapra, "Applied Numerical Methods with MATLAB® for Engineers and Scientists," 3rd edition, McGraw-Hill, New York/USA, 2012.
- [12] Richard L. Burden, and J. Douglas Faires, "Numerical Analysis", 9th edition, Brooks/Cole, Boston/USA, 2011.

